THE MECHANISM OF HEAT TRANSFER OF PARTICLES IN A FLUIDIZED BED

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The relationship between the contact and convective-conductive components of the heat flux in the heating of particles in a fluidized bed is examined.

In the conduction of various continuous processes in a fluidized bed the material fed into the bed is heated to the mean temperature of the bed. Each particle of material obtains heat by direct contact with other particles, by the convective-conductive transfer of heat through the gas interlayer, and by thermal radiation. In a number of cases where the bed temperature is low radiative heat transfer can be neglected. To elucidate the mechanism of heat transfer to a particle contained in a fluidized bed we consider the relationship between the contact and convective-conductive components of the heat flux. This relationship determines the heating of particles in the bed.

We consider an idealized model of a uniformly fluidized bed composed of smooth elastic spherical particles of the same radius. We will assume that each particle undergoes collision with only one particle at one time. Heat transfer in the case of brief contact of two bodies with constant temperature on the contact surface was examined in [1]. If the period of contact is sufficiently brief the bodies can be regarded as infinitely produced in the direction of the heat flux. In this case the temperature t_c on the contact surface during the time of contact will remain constant and on the surface of the two bodies on the side opposite their contact surface the temperatures will remain the same as the initial temperatures of each body (Fig. 1).

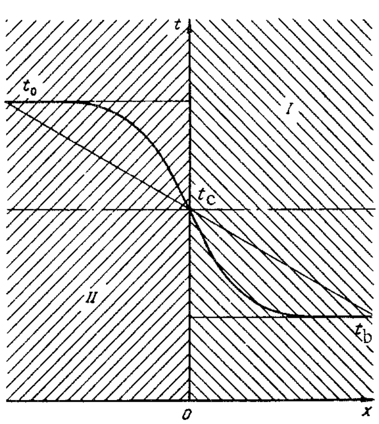

Fig. 1. Diagram illustrating heat transfer during the brief contact of two bodies (I and II) at $\tau = 0$.

The duration of contact for which the above conditions are fulfilled can be determined on the assumption that the maximum change of temperature on the side opposite the contact surface is 3%. In this case, as was shown in [1], the maximum duration of contact is

$$\tau_{c.\,max} = 381.5\, d^2/a. \qquad (1)$$

As the conducted calculations showed, τ_c is almost always $\ll \tau_{c.\,max}$.

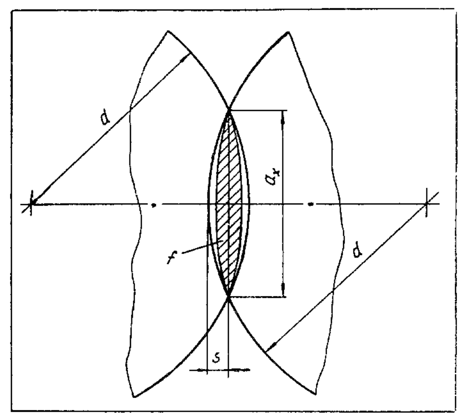

Fig. 2. Deformation of spherical particles on collision.

In the case of a central collision of spherical particles the contact surface is a plane and the region of contact is circular. The conditions of the problem are formulated thus: On the surface of a plane body with initial temperature t_0, the temperature increases instantaneously to t_c when $\tau = 0$. The amount of heat transferred during the time of contact τ_c of the two bodies has to be determined. The solution of the problem posed in [1] gives the following expression for the amount of heat transferred by the brief contact of two bodies:

$$Q_c = f\,\frac{2}{\sqrt{\pi}}\,\sqrt{\lambda_m c_m \gamma_m}\,\sqrt{\tau_c}\,(t_c - t_0). \qquad (2)$$

To compare the heat flux transferred by contact and the convective-conductive heat flux through the gas interlayer we will relate the amount of heat to the surface of the particle $F_p = \pi d^2$ and the time τ_f between two collisions:

$$q_c = 2f\sqrt{\lambda_m c_m \gamma_m} \times \sqrt{\tau_c}\,(t_c - t_0)/\sqrt{\pi}\,F_p \tau_f. \qquad (3)$$

If the bed is uniformly fluidized we can assume that the mean distance between the centers of the particles (in the case of cubic packing of the spheres) is [2]

$$l = 0.807\, d/(1-\varepsilon)^{1/3} = d\,(z+1). \qquad (4)$$

Then the time between collisions is expressed by:

$$\tau_f = (l-d)/w_p = dz/w_p. \qquad (5)$$

Table 1

Values of $m = q_c/q_g$ for Particles of Different Material and Size

Particle material	Particle diameter d, mm	Particle velocity, m/sec	Porosity ε of bed			
			0.5	0.6	0.7	0.8
			Values of m			
Glass	0.5	10^{-3}	$0.1015 \cdot 10^{-2}$	$0,828 \cdot 10^{-4}$	$0,193 \cdot 10^{-4}$	$0.611 \cdot 10^{-5}$
		10^{-2}	$0.51 \cdot 10^{-1}$	$0,41 \cdot 10^{-2}$	$0.891 \cdot 10^{-3}$	$0.246 \cdot 10^{-3}$
		10^{-1}	$0.256 \cdot 10^{1}$	$0,206 \cdot 10^{0}$	$0.46 \cdot 10^{-1}$	$0.123 \cdot 10^{-1}$
		10^{0}	$0,128 \cdot 10^{3}$	$0,103 \cdot 10^{2}$	$0,237 \cdot 10^{1}$	$0.614 \cdot 10^{0}$
"	1,0	10^{-3}	$0,11 \cdot 10^{-3}$	$0,151 \cdot 10^{-4}$	$0.565 \cdot 10^{-5}$	$0.276 \cdot 10^{-5}$
		10^{-2}	$0.534 \cdot 10^{-2}$	$0,815 \cdot 10^{-3}$	$0,264 \cdot 10^{-4}$	$0.97 \cdot 10^{-5}$
		10^{-1}	$0.27 \cdot 10^{0}$	$0,357 \cdot 10^{-1}$	$0,12 \cdot 10^{-1}$	$0.456 \cdot 10^{-2}$
		10^{0}	$0.134 \cdot 10^{2}$	$0,1795 \cdot 10^{1}$	$0.604 \cdot 10^{0}$	$0,23 \cdot 10^{0}$
"	2,0	10^{-3}	$0,286 \cdot 10^{-4}$	$0.397 \cdot 10^{-5}$	$0,16 \cdot 10^{-5}$	$0.825 \cdot 10^{-6}$
		10^{-2}	$0,1295 \cdot 10^{-2}$	$0.16 \cdot 10^{-3}$	$0,703 \cdot 10^{-4}$	$0.262 \cdot 10^{-4}$
		10^{-1}	$0,648 \cdot 10^{-1}$	$0,921 \cdot 10^{-2}$	$0,324 \cdot 10^{-2}$	$0,122 \cdot 10^{-2}$
		10^{0}	$0,323 \cdot 10^{1}$	$0,462 \cdot 10^{0}$	$0,162 \cdot 10^{0}$	$0,61 \cdot 10^{-1}$
Steel	1,0	10^{-3}	$0,778 \cdot 10^{-3}$	$0.105 \cdot 10^{-3}$	$0,401 \cdot 10^{-4}$	$0,184 \cdot 10^{-4}$
		10^{-2}	$0,359 \cdot 10^{-1}$	$0,5 \cdot 10^{-2}$	$0,178 \cdot 10^{-2}$	$0.694 \cdot 10^{-3}$
		10^{-1}	$0,18 \cdot 10^{1}$	$0.231 \cdot 10^{0}$	$0,887 \cdot 10^{-1}$	$0.337 \cdot 10^{-1}$
		10^{0}	$0,91 \cdot 10^{2}$	$0.126 \cdot 10^{2}$	$0,444 \cdot 10^{1}$	$0,169 \cdot 10^{1}$
Lead	1,0	10^{-3}	$0,194 \cdot 10^{-2}$	$0,297 \cdot 10^{-3}$	$0,129 \cdot 10^{-3}$	$0.788 \cdot 10^{-4}$
		10^{-2}	$0,95 \cdot 10^{-1}$	$0.132 \cdot 10^{-1}$	$0,472 \cdot 10^{-2}$	$0.188 \cdot 10^{-2}$
		10^{-1}	$0,477 \cdot 10^{1}$	$0.652 \cdot 10^{0}$	$0,229 \cdot 10^{0}$	$0.863 \cdot 10^{-1}$
		10^{0}	$0,238 \cdot 10^{3}$	$0,327 \cdot 10^{2}$	$0,114 \cdot 10^{2}$	$0.432 \cdot 10^{1}$

In the general case the temperature on the contact surface can be determined from the expression [3]

$$(t_b - t_c)/(t_c - t_0) = \sqrt{\lambda_1 c_1 \gamma_1} / \sqrt{\lambda_2 c_2 \gamma_2}, \quad (6)$$

where the subscripts 1 and 2 refer to the two contacting bodies. In this case the material of the two bodies is the same and, hence, we can write from (6)

$$t_c = (t_b + t_0)/2. \quad (7)$$

To determine the surface of contact f and the duration of the collision τ_c we use Hertz's theory of elastic collisions of spherical bodies, which is expounded in [4]. The basic assumptions of this theory—the elastic nature of the collisions and the identical nature of the static and dynamic interaction of the bodies—have been confirmed experimentally. For the collision of two spheres of the same radius and of the same material the time of contact is

$$\tau_c = 2.9432 \left[\frac{25\pi^2}{8} \frac{(1-\mu)^4}{(1-2\mu)^2} \right]^{1/5} \frac{d}{2w_f^{1/5} v^{4/5}}. \quad (8)$$

The velocity v of propagation of a compression wave in a material is equal to the velocity of sound in the material and can be determined from the expression

$$v = \sqrt{E(1-\mu)/\gamma_m(1+\mu)(1-2\mu)}. \quad (9)$$

The maximum deformation of the particles (half of the maximum linear approach of the particle centers) is

$$s_{\max} = \tau_к w_p / 2.9432. \quad (10)$$

The area of contact varies during the approach of the particles from 0 to f and corresponds at each instant to the deformation s of the particles (Fig. 2). The instantaneous diameter of the area of contact is determined from geometrical considerations:

$$a_x = 2\sqrt{sd - s^2}. \quad (11)$$

Hence, the instantaneous area of contact is

$$f_x = \pi a_x^2/4 = \pi (sd - s^2). \quad (12)$$

The mean area of contact during the time of collision τ_c of two particles is

$$f = \frac{1}{s_{\max}} \int_0^{s_{\max}} f_x dx = \pi \times \left(\frac{d s_{\max}}{2} - \frac{s_{\max}^2}{3} \right). \quad (13)$$

Since the deformation $s_{\max}$ of the particles is extremely small, the second term in the right side of expression (13) can be neglected, and then

$$f \approx \frac{\pi}{2} d s_{\max}. \quad (13a)$$

We put

$$B = \left[\frac{25\pi^2}{8} \frac{(1-\mu)^4}{(1-2\mu)^2} \right]^{1/5} \frac{1}{2v^{4/5}}, \quad k = 2.9432.$$

Then

$$\tau_c = kBd/w_p^{1/5}, \quad (8a)$$

$$f = \pi d^2 B w_p^{4/5}/2. \quad (13b)$$

Substituting the values of t_c, τ_c, and f from Eqs. (7), (8a), and (13b) in Eq. (3), we obtain

$$q_c = \frac{B^{3/2} w_p^{1.7} \sqrt{k \lambda_m c_m \gamma_m}}{2\sqrt{\pi d z}} (t_b - t_0). \quad (14)$$

Thus, the contact heat flux increases with reduction in Young's modulus E and particle diameter d, and with increase in particle velocity w_p and the coefficient of thermal activity $(\lambda_m c_m \gamma_m)^{1/2}$.

We consider the transfer of heat to the particle through the gas interlayer due to convection and molecular heat conduction of the gas. The equation of the heat balance of the particle can be written as

$$c_m \gamma_m \frac{\pi d^3}{6} dt = \alpha \pi d^2 (t_b - t) d\tau. \quad (15)$$

We separate the variables and integrate both sides of the equation

$$\int_{t_0}^{t} \frac{c_m \gamma_m d}{6\alpha} \frac{dt}{t_b - t} = \int_0^{\tau} d\tau. \quad (16)$$

Hence

$$t_b - t = (t_b - t_0) \exp(-b\tau), \quad (17)$$

where

$$b = 6\alpha / c_m \gamma_m d.$$

The mean heat flux during the period τ_f is

$$q_g = \frac{1}{\tau_f} \int_0^{\tau_f} \alpha (t_b - t) d\tau. \quad (18)$$

Substituting $(t_b - t)$ from Eq. (17) in Eq. (18) and integrating, we obtain

$$q_g = \frac{c_m \gamma_m d}{6\tau_f} [1 - \exp(-b\tau_f)] (t_b - t_0). \quad (19)$$

Putting $p = 6\alpha / c_m \gamma_m w_p$ we determine the ratio of the contact and convective-conductive heat fluxes by using expressions (14) and (19),

$$m = q_c / q_g = A w_p^{0.7} / [1 - \exp(-pz)], \quad (20)$$

where

$$A = 3B\sqrt{akB/d}.$$

From formula (20) we calculated the values of m for glass (d = 0.5, 1.0, 2.0 mm), steel (d = 1.0 mm), and lead (d = 1.0 mm) spherical particles for different values of porosity of the bed and particle velocity w_p. For each material and size of particle we determined the Re number corresponding to the given porosity from the interpolation formula [5]:

$$Re = Ar\,\varepsilon^{4.75} / (18 + 0.6\sqrt{Ar\,\varepsilon^{4.75}}). \quad (21)$$

To calculate the coefficient of heat transfer between the particles and gas for glass spheres of diameter

Table 2

Characteristics of Heat Transfer in Fluidized Bed

Material	d, mm	w_p, m/sec	f/F_p	τ_c, sec	f/f_0	$\varepsilon = 0.5$	$\varepsilon = 0.6$	$\varepsilon = 0.7$	$\varepsilon = 0.8$	Q_c/Q_{c_0}
						τ_c/τ_f, %				
Glass	0.5	10^{-3}	$2.22\cdot10^{-6}$	$6.52\cdot10^{-6}$	0.76	0.0652	0.0145	0.00652	0.00343	0.206
	”	10^{-2}	$1.39\cdot10^{-5}$	$4.08\cdot10^{-6}$	4.75	0.408	0.0908	0.0408	0.0215	1.66
	”	10^{-1}	$0.875\cdot10^{-4}$	$2.57\cdot10^{-6}$	28.7	2.57	0.57	0.257	0.135	13.1
	”	10^{0}	$0.555\cdot10^{-3}$	$1.63\cdot10^{-6}$	190.0	16.3	3.62	1.63	0.86	105.0
	1.0	10^{-3}	$2.22\cdot10^{-6}$	$13.05\cdot10^{-6}$	0.76	0.0652	0.0145	0.00652	0.00343	0.147
	”	10^{-2}	$1.39\cdot10^{-5}$	$8.15\cdot10^{-6}$	4.75	0.408	0.0908	0.0408	0.0215	1.18
	”	10^{-1}	$0.875\cdot10^{-4}$	$5.15\cdot10^{-6}$	28.7	2.57	0.57	0.257	0.135	9.3
	”	10^{0}	$0.555\cdot10^{-3}$	$3.27\cdot10^{-6}$	190.0	16.3	3.62	1.63	0.86	74.8
	2.0	10^{-3}	$2.22\cdot10^{-6}$	$26.1\cdot10^{-6}$	0.76	0.0652	0.0145	0.00652	0.00343	0.103
	”	10^{-2}	$1.39\cdot10^{-5}$	$16.3\cdot10^{-6}$	4.75	0.408	0.0908	0.0408	0.0215	0.83
	”	10^{-1}	$0.875\cdot10^{-4}$	$10.3\cdot10^{-6}$	28.7	2.57	0.57	0.257	0.135	6.55
	”	10^{0}	$0.555\cdot10^{-3}$	$6.55\cdot10^{-6}$	190.0	16.3	3.62	1.63	0.86	52.5
Steel	1.0	10^{-3}	$2.1\cdot10^{-6}$	$12.3\cdot10^{-6}$	0.835	0.0615	0.0137	0.00615	0.00323	0.06
	”	10^{-2}	$1.32\cdot10^{-5}$	$7.75\cdot10^{-6}$	5.25	0.387	0.086	0.0387	0.0204	0.475
	”	10^{-1}	$0.83\cdot10^{-4}$	$4.9\cdot10^{-6}$	33.2	2.45	0.545	0.245	0.129	3.82
	”	10^{0}	$0.52\cdot10^{-3}$	$3.09\cdot10^{-6}$	209.0	15.4	3.43	1.54	0.81	30.0
Lead	1.0	10^{-3}	$6.4\cdot10^{-6}$	$37.5\cdot10^{-6}$	0.395	0.187	0.0416	0.0187	0.00985	0.0305
	”	10^{-2}	$4.05\cdot10^{-5}$	$23.7\cdot10^{-6}$	2.49	1.19	0.265	0.119	0.0268	0.24
	”	10^{-1}	$2.55\cdot10^{-4}$	$15.0\cdot10^{-6}$	15.8	7.5	1.67	0.75	0.395	1.92
	”	10^{0}	$1.61\cdot10^{-3}$	$9.45\cdot10^{-6}$	99.0	47.3	10.5	4.73	2.48	15.1

0.5 mm we used the formula obtained in [6] and suitable for the region 5 < Re < 70:

$$Nu = 0.021\,Re^{1.4}. \qquad (22)$$

In the remaining cases the heat transfer coefficient α was calculated from the formula recommended by Vasanova and Syromyatnikov [7] for the region 40 < < Re < 500:

$$Nu = 0.316\,Re^{0.8}. \qquad (23)$$

The results of calculation for $w_p = 10^{-3}, 10^{-2}, 10^{-1}$, and 10^0 m/sec are given in Table 1.

The table shows that the value of $m = q_c/q_g$ for the same porosity increases from a value of the order 10^{-5} to 10^1 with increase in particle velocity. Hence, while the contact heat flux for a particle velocity of 1 mm/sec is an insignificant fraction of the convective-conductive flux, for a particle velocity of 1 m/sec the contact heat flux can in some cases be tens of times greater than q_g. Thus, the transfer of heat to particles with different velocities is effected by different mechanisms. It is convenient to determine some mean velocity of the particles in the bed for a given gas velocity w_g in order to establish the main mechanism of heat transfer in particular conditions.

With reduction in particle size, as Table 1 shows, the value of m increases. Hence, small particles are apparently heated mainly by contact with other particles. With increase in porosity for a given material (i.e., with increase in the gas velocity) the value of m decreases rapidly. For materials of low elasticity (lead, for instance) the fraction of heat transferred by contact is much higher than in the case of highly elastic materials (steel).

It should be noted that the obtained results differ from the results of experimental investigations of the heat condition of dispersed materials in vacuum and at atmospheric pressure [8–11]. In these investigations the experimental data showed that contact heat transfer in stationary heaps is negligibly small in comparison with other methods of heat transfer. However, in the case of a fluidized bed the distribution of the different kinds of heat fluxes is altered. First of all, in the case of sufficiently high particle velocities the area of the contact region is different from that in a stationary heap. The ratios f/f_0 of these areas are given in Table 2. The value of f_0 was calculated from the formula obtained in [10].

The table shows that the area of the contact region is a fluidized bed may be tens of times greater than the corresponding area in a stationary heap. In addition, heat transfer during the brief contact of spherical particles is of quite a different nature from that in the case of a steady-state heat flux. This is evident from the nature of the change of temperature in the contact region for these two cases (Fig. 1). The temperature gradient at the point of contact in a fluidized bed is evidently much greater.

We calculated the ratios of the amounts of heat transferred in the same interval τ_c in the case of brief contact (Q_c) and in the case of contact in a stationary heap Q_{c_0} (Table 2). It is obvious that at high particle velocities (especially for material of low conductivity) this ratio is much greater than 1. It should be noted that the data obtained for lead at high particle velocities and low porosities are less reliable, since the time of contact is comparable with the period between two collisions (Table 2). This will presumably reduce the validity of the assumption of constant temperature in the period τ_c for this case. In the majority of cases τ_c is a few per cent of τ_f and, hence, the assignment of the heat flux to the period between the collisions and the assumption of constant temperature in the period τ_c are quite permissible. In practice τ_c is a component part of the period τ_f, since the mean particle velocity is obtained by averaging not only over all the particles, but over the time too, i.e., the time of contact is included.

Thus, we can expect a different kind of heat transfer in a fluidized bed from that in a stationary heap—the role of contact heat transfer will be greater. This idea has already been expressed in [2]. The considerable effect of contact heat transfer is important in the consideration of the mechanism of heating of particles entering a fluidized bed and the mechanism of heat exchange of particles with surfaces.

NOTATION

t_0, t are initial and instantaneous temperature of particle, introduced into bed; t_b is the mean temperature bed; t_c is the temperature on contact surface; d is the diameter of particles; τ is time; τ_c is the time of contact; $\tau_{c.max}$ is the maximum time of contact; τ_f is the time between two collisions of particles; F_p is the area of surface of particle; c_m, γ_m, λ_m, a are the heat capacity, density, thermal conductivity, and thermal diffusivity of particle material; f_x, f are the instantaneous and mean area of region of contact, respectively; l is the distance between centers of particles; $z = 0.807/(1-\varepsilon)^{1/3} - 1$ is porosity function determining the distance between particles; w_p is the velocity of particles in bed; q_c, q_g are the contact and convective-conductive components of heat flux, respectively; E is the modulus of elasticity of first kind; μ is the Poisson's ratio; v is the velocity of propagation of compression wave in material; s, s_{max} are the instantaneous and maximum deformation of colliding particles; a_x is the instantaneous diameter of area of contact; α is the coefficient of heat transfer from particles to gas; ε is porosity of bed; $Re = w_g d/\nu$ is the Reynolds number; $Ar = \frac{gd^3}{\nu^2}\frac{\gamma_m}{\gamma_g}$ is the Archimides number; $Nu = \alpha d/\lambda_g$ is the Nusselt number; ν, λ_g, γ_g are the kinematic viscosity, thermal conductivity, and density of gas; w_g is the gas velocity; f_0 is the area of region of contact in stationary heap; Q_c, Q_{c_0} are the amounts of heat transferred in brief contact in fluidized bed and in a stationary heap.

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